Proofs to Grade

Instructions: Analyze the alleged proofs of the claims below and give one of three grades. Assign a grade of A (excellent) if the claim and proof are correct, even if the proof is not the simplest or the proof you would have given. Assign an F (failure) if the claim is incorrect, if the main idea of the proof is incorrect, or if most of the statements in it are incorrect. Assign a grade of C (partial credit) for a proof that is largely correct but contains one or two incorrect statements or justifications. Explain your grade. Tell what is incorrect (where applicable) and why.

1. Suppose \( m \) is an integer.
   
   **Claim**: If \( m^2 \) is odd, then \( m \) is odd.
   
   **"Proof"**: Assume \( m \) is odd. Then \( m = 2k + 1 \) for some integer \( k \). Therefore,
   
   \[
   m^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1,
   \]

   which is odd. Therefore, if \( m^2 \) is odd, then \( m \) is odd.

2. Suppose \( x \) is a positive real number.
   
   **Claim**: The sum of \( x \) and its reciprocal is greater than or equal to 2. That is,
   
   \[
   x + \frac{1}{x} \geq 2
   \]

   **"Proof"**: Multiplying by \( x \), we get \( x^2 + 1 \geq 2x \). By algebra,
   
   \[
   x^2 - 2x + 1 \geq 0 \quad \quad (x - 1)^2 \geq 0.
   \]

   Any real number squared is greater than or equal to zero, so \( x + \frac{1}{x} \geq 2 \) is true.